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Geodetic applications of laser ranging

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The measurement of intersite distances with laser ranging to satellites has been demonstrated during the last few years for distances of several hundred to several thousand kilometres with precisions of a few tens of centimetres. These techniques are now being tested across the San Andreas fault in California where it is hoped plate motion will be observable after several years of measurements. The first measurements, between sites in southern and northern California, were made in 1972 and repeated again in 1974 with agreement between the baselines for each of the two years at the 10 cm level. The next measurements are planned for the summer of 1976. The results of these and related experiments will be described together with simulations of the projected capability using the high altitude Lageos satellite. General plans for future experiments will be described.

INTRODUCTION

Precision tracking of satellites at the centimetre level is presently being achieved by N.A.S.A. Goddard Space Flight Center laser systems. In addition, our ability to determine very accurate orbits from these data is steadily improving as our knowledge of, and our ability to model, the perturbing forces that act on the satellite increases. Presently our orbit determination capability is measured at the tens of centimetres level for the shorter orbital arcs with a deteriorating capability that reaches into the metres as the orbital arc increases from a few hours up to days.

Satellite geodesy utilizes satellite tracking as a means to determine the relative positions of points on the Earth's surface and can employ both dynamic and geometric methods. In the dynamic technique the oribital motion of the satellite is used in the solution and enables tracking station locations to be determined from a combination of the observations and the orbital theory. Either the observations or the theory can limit the accuracy of the ultimate result but information about a tracking station's location with respect to the centre of mass of the Earth can be obtained even when only one station is tracking. The geometric technique only uses simultaneous tracking data and requires little or no knowledge of the orbit but cannot be effectively applied to less than a network of six stations. The only factors which affect the accuracy of the solution are the measurement accuracy and measurement distribution. The only factors which affect the implementation of the technique are the requirements forced by simultaneity; satellite intervisibility between at least six stations, and certain restrictions on the relative positions of the stations that tend to make the geometry ill-conditioned.

Experience so far with range measurements has been exclusively with the dynamical approach, largely because insufficient laser tracking systems are presently available, and also because present satellite altitudes are too low for effective simultaneous tracking from more than three or four sites. Later this year N.A.S.A. plans to launch the Laser Geodynamics Satellite (Lageos) into a high-altitude orbit (5900 km) with inclination 110° (launched on 4 May 1976). This spacecraft

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should significantly improve the orbit determination capability for the dynamic analyses and make easier the acquisition of simultaneous data for testing the geometric approach.

In the present paper the latest results by using the dynamic methods on a low altitude satellite are described together with proposed experiments using both dynamic and geometric methods on the Lageos satellite.

Dynamic methods

Orbit determination of a satellite provides the motion of the satellite with respect to the centre of mass of the Earth. Thus, when the location of a tracking station is determined using the dynamic method the coordinates are with respect to the Earth's centre (mass). The simplest form of geodetic measurement is the estimation of the location of a tracking station's own coordinates – i.e. a single station analysis. In such a situation only a limited amount of information can be obtained from the tracking data. The longitude of the station is indeterminate and station latitude is only weakly resolvable since the latitude is generally highly correlated with the orbital inclination (variations in latitude are possible (Kolenkiewicz *et al.* 1976)). The third coordinate, height or radial distance from the Earth's centre, is measurable. If a station observes a satellite on several orbits the orbit can be determined and an adjustment of the vertical coordinate of the station made simultaneously. The quality of the height measurement is, of course, very dependent on the precision of the tracking data but more important is the ability to determine an accurate orbit. The most important factor in the orbit determination is the state of knowledge of the Earth's gravitational field. If the gravity forces are known perfectly then the height determination is of comparable accuracy to the range measurement but if the gravity field is poorly modelled in the orbit determination process, then errors in the model will dominate.

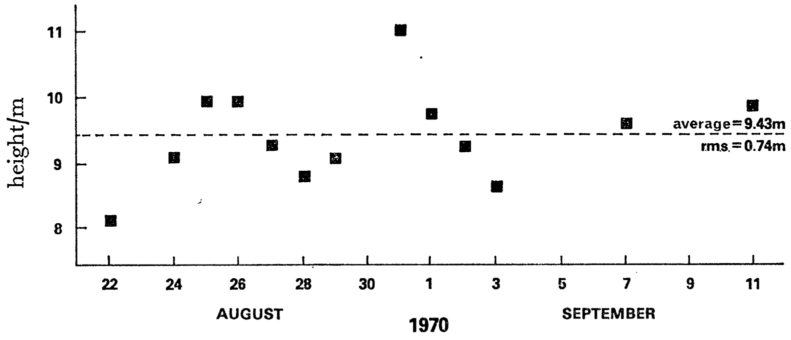

Figure 1. Spheroid height of laser at Greenbelt, Maryland.

An experiment with three weeks of laser data on the Beacon Explorer C spacecraft has been conducted in which thirteen measurements of the station height were obtained. An orbit was determined over a 3-week period from data obtained by a laser system at Goddard Space Flight Centre in Greenbelt, Maryland. By using this orbit as the trajectory of the spacecraft, the vertical position of the tracking station was adjusted on each of thirteen days by a least squares adjustment of the range measurements to the orbit. The heights obtained in this experiment are shown in figure 1. An individual measurement, which is based on four passes of the satellite covering a 6 h period, has a standard deviation about the mean of 74 cm (Dunn, Smith & Kolenkiewicz 1976). The individual heights are not distributed about the mean in figure 1 and therefore contain significant orbital error. The mean value of the spheroid height of 9.4 m implies a mean equatorial

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Earth radius of approximately 6378137 m for a value of $GM = 3.986006 \times 10^5\,\mathrm{km^3/s^2}$ when the geoid and the orbit are computed using the Goddard Earth Model 7 (GEM 7) (Wagner *et al.* 1976). If there are no additional systematic effects entering into the computations (which is unlikely to be true) then the radial distance of the station from the centre of the Earth is accurate to approximately 20 cm (standard deviation of the mean) from the 3 weeks of observations. Although this is barely of geophysical quality it shows that a single station can, in principle, monitor the larger vertical motions from year-to-year that occur at certain points on the Earth due to subsidence, for example.

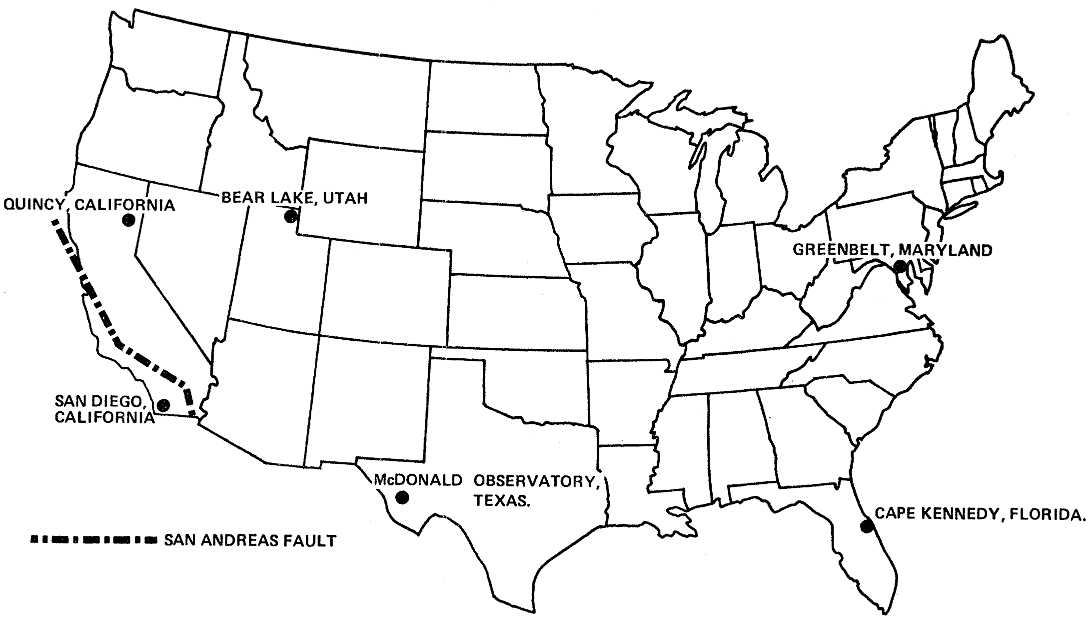

FIGURE 2. Laser tracking sites in the continental United States. The laser at McDonald Observatory is the lunar laser system; the laser near Cape Kennedy is the Patrick Air Force Base system. The lasers at Greenbelt, Bear Lake, Quincy and Utah are part of the San Andreas Fault Experiment.

With two stations in operation and able to track a satellite at the same time it is possible to infer the relative location of one station with respect to the other. Initial experiments were conducted in 1970 in which a 400 km baseline was measured with a precision of approximately 30 cm (Smith, Kolenkiewicz, Agreen & Dunn 1973). In 1972 a 900 km baseline was measured in California as a preliminary measurement in the San Andreas fault experiment (SAFE) and repeated in 1974 when an additional line of 3600 km was measured across the continental United States. These SAFE sites, San Diego and Quincy in California, and Greenbelt in Maryland are shown in figure 2. The San Andreas fault experiment is attempting to measure the motion along the fault in California by measuring the change in distance between sites on opposite sides of the fault. This motion is believed to be of the order of 4–5 cm per year between the San Diego and Quincy sites and the experiment requires the maximum accuracy that can be achieved. The technique that we have developed to measure these distances uses short orbital arcs of a few hours in which a satellite is tracked simultaneously from the two (or three) sites on several consecutive passes of the satellite near the stations. The primary satellite is the Beacon Explorer C spacecraft in a near circular orbit of 1000 km altitude and an orbital inclination of 41°. Because the latitudes of the tracking stations are comparable to the inclination of the orbit each station sees four or five

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consecutive passes of the satellite each day. When two (or more) stations observe the same three, four or five consecutive passes we are able to use those observations to determine the relative locations of the site(s).

The orbit and station coordinates are determined from only the tracking data collected at the sites in the experiment; no data from other tracking sites are used (even if available). The primary reason for not using data from other sources is that although additional data might be useful for determining the orbit, these observations can also introduce errors from uncertainties in the locations of the additional tracking stations. In our solutions, we solve for each parameter that we believe is a serious source of error and try to restrict the number of parameters to a minimum. In a two-station solution we solve for the orbit of the satellite (six parameters), the three coordinates of one tracking station, and a single coordinate, height, of the other station. For a three-station solution we solve for all the parameters of the two-station solution plus all the coordinates of the third station. Thus, only two station coordinates are held fixed – the latitude and longitude of the 'home' or 'base' station. Generally, no adjustment is made for parameters in the gravity field, drag forces, tides, mass of the Earth, etc. Although our solution is sensitive to errors in these parameters our data is unable to provide improved values. In our earlier solutions (Smith *et al.* 1973) we usually kept all coordinates at the base station fixed and only recovered the position of the second station but a consequence of this approach was a tendency to increase our sensitivity to errors in *GM*, the product of the gravitational constant and the Earth's mass. By also solving for the height at the base station we have found that the baselines that we recovered, particularly the shorter ones, were largely independent of *GM*.

Table 1. SAFE 1972 and 1974 station coordinate results

station	year	laser system	latitude (N)	longitude (E)	height (m)
San Diego	1972	Moblas 1	unadjusted	unadjusted	985.87 †
San Diego	1974	Moblas 2	unadjusted	unadjusted	985.87 †
Quincy	1972	Exper Laser	39° 58′ 24.470″	239° 03′ 37.624″	1059.44
Quincy	1974	Moblas 1	39° 58′ 24.474″	239° 03′ 37.650″	1060.36
Greenbelt	1974	Stalas	39° 01′ 13.161″	283° 10′ 19.806″	16.24

Coordinates of San Diego 32° 36′ 02.533″ N, 243° 09′ 32.864″ E.
Gem 8.
† Constrained to be equal.

San Diego is the base station for the San Andreas fault experiment, and two measurements of the San Diego–Quincy baseline have been made; one in 1972 and another in 1974. In 1972 a preliminary test was performed with 50 cm laser tracking systems, to test the concept of the experiment on a 900 km baseline. In 1974 the two sites were reoccupied for about 3 months with greatly improved systems of 10 cm quality. From the data of each of the years the three coordinates of Quincy and the height at San Diego have been recovered. In addition, the coordinates of a third station of similar quality operating from Greenbelt, Maryland have been recovered. Although the distance from San Diego to Greenbelt is over 3600 km, enough simultaneous tracking data was obtained on the BE-C satellite to provide a strong determination of the relative coordinates. Incidentally, none of the simultaneous observations between these two sites were above 20° elevation. Table 1 shows the solutions for the coordinates obtained in the 1972 and 1974 SAFE programme. These results were obtained in a joint solution for both years in which the height of San Diego was constrained to be the same for both years. Perhaps the most important

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features in table 1 are the close agreement in latitude between the 2 years and the modest disagreement in longitude and height. Since Quincy is nearly north of San Diego, the latitude closely represents the baseline and this result typifies our experience over the last few years: baselines are more strongly determined than distances perpendicular to the baseline.

Table 2. SAFE 1972 and 1974 baseline results

baseline	year	intersite distance m	internal precision cm	data quality cm	data fit cm
San Diego–Quincy	1972	896275.92	10	50	110
San Diego–Quincy	1974	896275.83	7	10	38
San Diego–Greenbelt	1974	3606114.80	7	10	83
Quincy–Greenbelt	1972	3702963.57	9	—	—
Quincy–Greenbelt	1974	3702963.23	8	—	—

GEM 8.

Table 2 shows the baselines between the sites in the SAFE 1972 and 1974 solutions. The agreement between the two San Diego to Quincy measurements is very close. The last three columns in table 2 have been included to provide an indication of the ultimate strength of these measurements. The internal precision is the precision of the baseline measurement based on the data fit shown in the last column; thus the agreement between the 1972 and 1974 baselines is within the effective noise level of the data. The 'data quality' column is an estimate of the actual accuracy of a single laser range measurement in the solution and indicates the data fit that can be ultimately expected when the model errors, such as gravity, in the solution are reduced to zero. Because the internal precision is proportional to the data fit we can anticipate that the accuracies of the baselines will improve to about 4 cm and 2 cm in 1972 and 1974, respectively, for the San Diego–Quincy line. There is no entry in the data columns for the Quincy–Greenbelt baseline because this result was inferred from the determinations of the San Diego–Quincy and San Diego–Greenbelt baselines rather than from the data. Insufficient simultaneous tracking was acquired between Quincy and Greenbelt for an independent estimate of this baseline. The results given in tables 1 and 2 were obtained with the GEM 8 gravity model (Wagner, Lerch, Brownd & Richardson 1976) which has $GM = 3.986008 \times 10^{14}\ \mathrm{m^3/s^2}$ and the Earth's mean equatorial radius as 6 378 145 m.

A simulation of the San Andreas fault experiment for the baseline between San Diego and Quincy (Agreen & Smith 1974) shows the major sources of error as the gravity field, *GM* and the height of the San Diego station. In this simulation no adjustment was made to the San Diego height. In summary, this simulation suggested that a precision of about 2 cm was possible by the end of this decade and assumed the BE-C spacecraft was the primary spacecraft. This conclusion was largely based on an assumed improvement in our knowledge of the gravity field of the Earth of a factor of about 3 between 1975 and 1980. If this goal is unrealized then the projected results would be accordingly affected. However, it fails to take into account any improvement that could come about by making use of the Laser Geodynamics Satellite (Lageos) which is due for launch soon. Lageos will be placed into a high altitude orbit of about 6000 km and will be much less affected by gravity model errors than BE-C. Even if no improvement in the gravity model takes place, the SAFE baseline measurements should eventually be measurable to an accuracy of 1 or 2 cm with Lageos. Furthermore, because its altitude is so great it will be much easier to obtain baseline measurements of continental scale than is now possible with satellites such as BE-C.

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During the Summer of 1976 the SAFE tracking sites of San Diego, Quincy and Greenbelt will again track BE-C for a re-determination of the station locations. In addition, a site near Bear Lake, Utah will be occupied at the same time and extend the SAFE network to four stations. The Bear Lake site was chosen because we hope to be able to eventually determine if spreading is taking place across the basin and range province between Quincy, California and Bear Lake, Utah. Projected spreading rates are extremely small, at much less than 1 cm/year, so accurate measurements over a long period will be required. Two other stations expect to be operating during this same period from the continental United States, the Smithsonian Astrophysical Observatory's laser at Mt Hopkins, Arizona and a system at Patrick Air Force Base in Florida. If all six systems do satisfactorily operate during this period we are planning to determine the locations of all the sites, using San Diego as the base station. Successful operation over a two or three month period should provide geodetic ties between all the stations at the one or two tens of centimetre level, based on our present experience. For the primary line, between San Diego and Quincy, this will be the third determination and the second with lasers of 10 cm quality or better.

TABLE 3. SIMULATION OF GEOMETRIC SOLUTION FOR SIX STATIONS (LAGEOS)

		magnification factor		
baseline	length km	12 obs	24 obs	36 obs
Quincy, Ca.–Bear Lake, Ut.	831	13	4	3
San Diego, Ca.–Quincy, Ca.	896	10	3	2
McDonald Obs., Tx–San Diego, Ca.	1244	21	7	6
Greenbelt, Md.–Cape Kennedy, Fl.	1246	17	5	4
Cape Kennedy, Fl.–McDonald obs., Tx	2237	44	14	11
Bear Lake, Ut.–Greenbelt, Md.	2893	56	18	14

GEOMETRIC METHODS

With six tracking stations expected to be operating in the summer of 1976 a geometric solution of the relative station positions is, in principle, feasible. Geometric solutions require exactly simultaneous measurements from several sites on several different occasions but require no knowledge of the orbit and, indeed, need not be on the same satellite. The accuracy of the coordinates only depends on the accuracy of the range measurements and the strength of the geometry. The difficulties with the method are largely in the acquisition of the simultaneous data from several sites.

In principle, simultaneous tracking from only four stations is required for a geometric solution, but because of singularities in the solution, five on preferably six stations are required. This restriction has been discussed extensively by Escobal *et al.* (1973). Our experience during the 1974 SAFE observation period indicates that a laser station is successful in tracking on approximately 25 % of the occasions when weather, system breakdown, computer failure, etc., are all taken into account. Thus the probability of six stations observing simultaneously is approximately $(0.25)^6$, or about 0.04 %. Eventually the efficiency of laser tracking will probably reach 50 % but this still only indicates a 2 % chance for simultaneity from six stations.

With six stations operating in 1976 we hope to be able to test some aspects of the geometric method. With satellites presently in orbit the chances of simultaneous tracking from all six sites are even more remote. At 1000 km the only region of the orbit that could be simultaneously visible from all sites is over the central United States and from most sites these positions will have

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elevations below 20°. If, however, four or more simultaneous events are obtained we shall be able to test the geometric method.

In order to assess the value of the geometric method a number of simulations have been performed on a network very similar to the one expected to be tracking this summer. This network is shown in figure 2. The simulations assume all six stations track the Lageos satellite at 6000 km and the results shown are the accuracies of the baselines given as a dimensionless magnification factor based on an observation of unit weight. Thus, if the range measurements have an accuracy of 2 cm, the accuracy of a particular baseline (in centimetres) is the magnification factor multiplied by 2. Table 3 shows the results of this simulation for a selection of baselines. The number of observations shown in table 3 is the number of simultaneous events and each observation is composed of six simultaneous range measurements.

The most important conclusion from table 3 is that all magnification factors are greater than one, even for 36 observations, and for the baselines to be as well determined as a range measurement at least 100 observations would be required even for the most favourable baselines. Furthermore, there is a clear tendency for the magnification factor to get larger as we go down table 2 and this is because the baselines are ordered according to increasing length. This result, a larger magnification factor for a longer baseline, makes the measurement of very long baselines extremely difficult if magnification factors near to unity are required. At the present time we believe our laser systems are accurate to about 5 cm and therefore our projected ability to measure the lines shown in table 2 is about 10 cm for the shorter lines and 70 cm for the longest line, based on 36 observations. The difficult question to answer is how long it would take to obtain these observations since we have no valid experience. If we make the assumption that all 36 observations were obtained on a single pass of Lageos tracked by all stations, and that there are four possible passes per day, then it would take only about 16 days if each station were 50 % efficient – but 1024 days if only 25 % efficient. Thus, a high efficiency is critical for a large number of stations to successfully complete a geometric solution.

The magnification factor can be reduced below that shown in table 3 by having the observations taken on two satellites at different altitudes. If a few of the 36 observations of Lageos are actually obtained on a much lower satellite at 1000 km altitude then a significant decrease in the magnification factor can be achieved. Simulations indicate a factor of 2 is possible, but it must be remembered that simultaneous observation of a low altitude satellite is much more difficult than a high satellite and would take correspondingly longer. Thus table 2, although not necessarily representing the most favourable situation, does indicate the overall capability and limitations of realistic geometric solution.

Discussion

Although laser ranging to near Earth satellites began in the mid-1960s it is only in its infancy with respect to its application to geodesy and geophysics. Our experience so far at N.A.S.A. has been restricted to at most three systems operating simultaneously, and consequently to the dynamic methods. Although the measurement of global distances may not permit the use of geometric techniques they may be applicable to regions, such as North America, where Lageos can be easily observed from both sides of the continent at the same time. Further, the geometric technique is clearly better suited to shorter baselines than longer ones (table 3). On the other hand, the dynamic method can be used on any number of stations, does not require any degree of true simultaneity and is projected to have at least the same capability as the geometric method

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if the problems of modelling the spacecraft motion can be solved. For Lageos this may well be adequately realizable.

Although laser tracking of satellites is expected to give high accuracy distance measurement between specific points on the Earth's surface it is not a general purpose surveying instrument. For the very high density, high accuracy surveying of the future an alternative approach is currently being developed that will utilize the Space Shuttle. In this concept the laser system is in the Shuttle and the corner cubes are on the ground. A large number of cubes, spaced 5–25 km apart, are spread over an area of special geodetic or geophysical interest and the laser in space ranges down to each of the cubes, several at a time, as the Shuttle moves in its orbit. After approximately one week it is anticipated that the relative locations of the cubes on the ground will be known to approximately 1 cm in each coordinate (Vonbun, *et al.* 1975; Mueller, *et al.* 1975). No special accuracy is required of the laser or the orbit of the Shuttle since the technique utilizes a range differencing between cubes in the same laser beam. Measurement of the locations every 3 months will reveal the crustal motions on a scale of the cube spacing. In order for the system to be located with respect to the centre-of-mass of the Earth the ground based network of laser reflectors will probably be located in a region bounded by conventional laser tracking systems that are tracking Lageos and thus continually monitoring their own centre-of-mass position.

Laser ranging to or from near Earth satellites in the future looks as if it will be addressing measurements on scales of 5–5000 km with projected accuracies at the few centimetre level. Much needs to be learned about the methods and their application but the technology appears to be here, or just around the corner, and the future looks bright and exciting.

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